Problems from Chapters 10-13, due Monday, March 30th at beginning of class. You may work together in groups.

1. Use the pumping lemma to show that each of these languages is non-regular:
   a. \( \{a^n b^{n+1}\} = \{abb aabbb aaabbbb \ldots\} \)
   b. \( \{a^n b^n a^n\} = \{aba aabaa aabbbaaa \ldots\} \)
   c. \( \text{PRIME} = \{a^p \text{ where } p \text{ is any prime}\} = \{aa aaaa aaaaa \ldots\} \)

2. Use the Myhill-Nerode theorem to show that each of these languages (same as #1) is non-regular:
   a. \( \{a^n b^{n+1}\} = \{abb aabbb aaabbbb \ldots\} \)
   b. \( \{a^n b^n a^n\} = \{aba aabaa aabbbaaa \ldots\} \)
   c. \( \text{PRIME} = \{a^p \text{ where } p \text{ is any prime}\} = \{aa aaaa aaaaa \ldots\} \)

3. The language MOREA is defined as follows:
   \( \text{MOREA} = \{\text{all strings of a's and b's in which the total number of a's is greater than the total number of b's}\} = \{a a aab aba baa aaab aaba \ldots\} \)
   a. Use the fact that \( \text{MOREA}' \cap \text{MOREB}' \cap (a + b)^* = \text{EQUAL} \)
      To prove that MOREA is nonregular (where MOREB has its obvious meaning)
   b. Explain why the pumping lemma cannot be used to prove that MOREA is nonregular
   c. Show that MOREA can be shown to be nonregular by the Myhill-Nerode theorem
      by using the sequence
      \[ \text{aab aaab aaaaab aaaaaab} \ldots \]

4. Consider the CFG:
   \[
   S \rightarrow XaXaX \\
   X \rightarrow aX | bX | \lambda
   \]
   What is the language that this CFG generates?

5. Consider the CFG:
   \[
   S \rightarrow XbaaX \mid aX \\
   X \rightarrow Xa | Xb | \lambda
   \]
   What is the language that this CFG generates? Find a word in this language that can be generated in two substantially different ways (show the ways)

6. Find CFGs for the following languages over the alphabet \( \Sigma = \{a, b\} \):
   a. All words in which the letter b is never tripled
   b. All words that do not have the substring ab
   c. MOREA (all strings with more a’s than b’s)

7. Show that the following CFGs are ambiguous:
   a. \( S \rightarrow SaSaS \mid b \)
   b. \( S \rightarrow aS \mid aSb \mid X \\
      X \rightarrow aX \mid a \)

8. Begin to draw the total language tree for the following CFG until you have found all words of at least one, two, or three letters:
   \[
   S \rightarrow aS \mid bS \mid a
   \]
9. Given the grammar below, eliminate the $\lambda$-productions using the procedure discussed in class and provide a grammar that generates the same language.
   \[ S \rightarrow XaX \mid bX \]
   \[ X \rightarrow XaX \mid XbX \mid \lambda \]

10. The following CFG has unit production(s). Using the algorithm presented in class, find a CFG for the same language that does not have unit productions.
   \[ S \rightarrow AA \]
   \[ A \rightarrow B \mid BB \]
   \[ B \rightarrow abB \mid b \mid bb \]

11. Convert the following CFGs to CNF:
   a. \[ S \rightarrow ABABAB \]
      \[ A \rightarrow a \mid \lambda \]
      \[ B \rightarrow b \mid \lambda \]
   b. \[ S \rightarrow SS \mid A \]
      \[ A \rightarrow AA \mid AS \mid a \]

12. Find the leftmost derivation for the word $abba$ in the grammar (this cannot be done as a tree!)
   \[ S \rightarrow AA \]
   \[ A \rightarrow aB \]
   \[ B \rightarrow bB \mid \lambda \]